14 a) $\sum\limits_{n=1}^{\infty} n^2 x^n$

použijeme podílové kritérium:

$$\lim_{n\to\infty} \frac{(n+1)^2\, x \cdot \cancel{x^n}}{n^2\, \cancel{x^n}} = \underbrace{\frac{(n+1)^2}{n^2}}_{\to 1} \cdot x < 1$$

$\Rightarrow$ $\underline{R = 1}$ v bodech $\underline{\underline{\pm 1 \text{ diverguje}}}$

---

14 b) $\sum\limits_{n=1}^{\infty} \frac{x^n}{n!}$

podílové kritérium: $$\lim_{n\to\infty} \frac{\frac{x\cdot \cancel{x^n}}{\cancel{n!}\cdot (n+1)}}{\frac{\cancel{x^n}}{\cancel{n!}}} = \frac{x}{(n+1)} < 1$$

$\Rightarrow$ $\underline{\underline{R = \infty}}$

---

14 c) $\sum\limits_{n=1}^{\infty} (n+1)!\, x^n$

podílové kritérium: $$\lim_{n\to\infty} \frac{(n+2)\cdot \cancel{(n+1)!} \cdot x \cdot \cancel{x^n}}{\cancel{(n+1)!}\ \cancel{x^n}} = \underbrace{(n+2)}_{\to\infty} \cdot x < 1$$

$\Rightarrow$ $\underline{\underline{R = \emptyset}}$

15 a) $\sum_{n=1}^{\infty} \frac{(3n)!}{n^n (2n)!} x^n$

podílové kritérium:

$$\lim_{n\to\infty} \frac{\frac{(3n+3)\cdot(3n+2)\cdot(3n+1)\cdot(3n)!\ \cancel{x}\cdot\cancel{(n+1)}^{n}}{(2n+2)\cdot(2n+1)\cdot(2n)!\ n\cdot(n+1)^n}}{\frac{\cancel{(3n)!}\ \cancel{x}^n}{\cancel{(2n)!}\ n^n}} =$$

$$= \lim_{n\to\infty} \underbrace{\frac{27n^3 \ldots}{4n^3 \ldots}}_{\to \frac{27}{4}} \cdot \underbrace{\frac{n^n}{(n+1)^n}}_{\to \frac{1}{e}} \cdot x$$

$$\Rightarrow \quad \frac{27}{4e} x < 1$$

$$x < \frac{4e}{27}$$

$$\Rightarrow \quad R = \frac{4e}{27} \qquad S = \emptyset$$

---

15 b) $\sum_{n=1}^{\infty} \frac{n!\,(x-2)^n}{n^n}$

podílové krit: $$\lim_{n\to\infty} \frac{\frac{\cancel{(n+1)}\,\cancel{n!}\cdot(x-2)\cdot\cancel{(x-2)^n}}{(n+1)^{\cancel{(n+1)}}}}{\frac{\cancel{n!}\,\cancel{(x-2)^n}}{n^n}} = \lim \underbrace{\frac{n^n}{(n+1)^n}}_{\to \frac{1}{e}} \cdot (x-2)$$

$$\Rightarrow \quad \frac{(x-2)}{e} < 1 \qquad \Rightarrow \quad R = e$$

$$(x-2) < e \qquad\qquad S = \emptyset$$

SIP

3 a) $y'' - y = 2e^x - x^2$

řešení homogenní rovnice:

$$\lambda^2 - 1 = 0 \quad \Rightarrow \text{ řešení homogenní rovnice je:}$$
$$\underline{\lambda_{1,2} = \pm 1}$$
$$y = C_1 \cdot e^x + C_2 \cdot e^{-x}$$

nehomogenní rovnice I.

$$y_1 = ax^2 + bx + c$$
$$y_1' = 2ax + b$$
$$y_1'' = 2a \qquad \Rightarrow \quad 2a - (ax^2 + bx + c) = -x^2$$

$$x^2: \quad -a = -1 \quad \Rightarrow \quad \underline{a = 1}$$
$$x^1: \quad -b = 0 \quad \Rightarrow \quad \underline{b = 0}$$
$$x^0: \quad 2a - c = 0 \quad \Rightarrow \quad \underline{c = 2}$$

$$\boxed{y_1 = x^2 + 2}$$

nehomogenní rovnice II.

$$y_2 = k \cdot x \cdot e^x$$
$$y_2' = k \cdot e^x + k \cdot x \cdot e^x$$
$$y_2'' = k \cdot e^x + k \cdot e^x + k \cdot x \cdot e^x$$

$$\Rightarrow \quad k \cdot e^x + k \cdot e^x + \cancel{k \cdot x \cdot e^x} - \cancel{k \cdot x \cdot e^x} = 2e^x$$
$$\cancel{2}k \cdot \cancel{e^x} = \cancel{2e^x}$$
$$\underline{k = 1}$$

$$\boxed{y_2 = x e^x}$$

řešení homogenní rovnice: $\underline{y = C_1 e^x + C_2 e^{-x} + x \cdot e^x + x^2 + 2}$

② a) $x' - x \cotg t = \sin t$ pp: $x(\frac{\pi}{2}) = \varphi$

řešení homogenní:

$$\frac{dx}{dt} = x \cotg t$$

$$\frac{1}{x} dx = dt \cotg t$$

$$\frac{1}{x} dx = \frac{\cos t}{\sin t} dt \quad \left/ \begin{array}{l} \text{subst } \sin t = u \\ \cos t\, dt = du \end{array} \right/$$

$$\int \frac{1}{x} dx = \int \frac{1}{u} du$$

$$\ln|x| = \ln|\sin t| + \ln|C|$$

$$\underline{\underline{x = C \cdot \sin t}}$$

⇓

homogenní rovnice / variace konstanty

$$x = u(x) \sin t$$

$$x' = u(x)' \sin t + u(x) \cdot \cos t$$

⇓

dosazení do původní rovnice:

$$(u(x)' \cdot \sin t) + \cancel{(u(x) \cos t)} - \cancel{(u(x) \sin t \cdot \frac{\cos t}{\sin t})} = \sin t$$

$$u(x)' \cdot \sin t = \sin t$$

$$u(x)' = 1$$

$$\underline{\underline{u(x) = t + K}}$$

dosazení, tedy obecné řešení: $x = (t + K) \cdot \sin t$

PP: $0 = (\frac{\pi}{2} + K) \cdot 1$

$K = -\frac{\pi}{2}$ ⇒ řešení počáteční úlohy.

$$\underline{\underline{x = (t - \frac{\pi}{2}) \cdot \sin t}}$$

MARTIN @ LIPINSKY.CZ

SIP

① a) $x = x' \cos^2 t \ln x$ PP: $x(\pi) = 1$

$$x = \frac{dx}{dt} \cos^2 t \ln x$$

$$\int \frac{1}{\cos^2 t}\,dt = \int \frac{\ln x}{x}\,dx$$

$\boxed{\text{subst } \ln x = u,\quad \tfrac{1}{x}dx = du}$

$$\operatorname{tg} t = \int u\,du$$

$$\operatorname{tg} t = \frac{u^2}{2}$$

$$\operatorname{tg} t = \frac{\ln^2 x}{2} + C$$

$$2\operatorname{tg} t - \ln^2 x + C = 0$$

$$\underbrace{2\cdot\operatorname{tg}\pi}_{0} - \underbrace{\ln^2 1}_{0} + C = 0$$

$$C = 0$$

⇒ řešení počáteční úlohy je:

$$2\operatorname{tg} t - \ln^2 x = 0$$

PP: $x(\frac{\pi}{2}) = -1$

① b) $x\,dt = \operatorname{cotg} t\,dx$

$$\frac{1}{\operatorname{cotg} t}\,dt = \frac{1}{x}\,dx$$

$$\operatorname{tg} t\,dt = \frac{1}{x}\,dx$$

| subst $\cos t = u$, $\sin t\,dt = du$ $\quad \int \frac{1}{u}\,du = \ln|u| + C$ |

$$\int \frac{\sin t}{\cos t}\,dt = \int \frac{1}{x}\,dx$$

[dosazení počáteční podmínky

$$\ln|\cos t| = \ln|x| + \ln C \qquad \cos\frac{\pi}{2} = -1\cdot C$$

$$\cos t = x\cdot C \qquad 0 = C$$

$$\Rightarrow \quad x = \cos t\cdot(-1) \Rightarrow x = -\cos t$$

nezjištěné :(